Evaluation of the Humboldt GeoGauge^{TM} on Dry Cohesionless Silica Sand in a Cubical Test Bin

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Introduction

The GeoGauge™ manufactured by the Humboldt Manufacturing Company is a portable instrument providing a simple and rapid means of measuring the stiffness of compacted subgrade, subbase, and base course layers in earthen construction. The GeoGauge™ measures stiffness at the soil surface by imparting very small displacements to the soil on an annularly loaded region via a harmonic oscillator operating over a frequency of 100 to 196 Hz. Appropriate transducer technology is incorporated to measure both force and displacement from which stiffness can be computed. The computed stiffness is determined based on an average of 25 stiffness values obtained at 25 discreet frequencies in the frequency band cited above.

The GeoGauge™ weighs approximately 10 kg (22 lb). The annular ring which contacts the soil has an outside diameter of 4.50 in. (114 mm) and an inside diameter of 3.50 in. (89 mm); hence, with a ring thickness of 0.50 in. (13 mm). Humboldt claims that the magnitude of the vertical displacement induced at the soil-ring interface is less than 0.00005 in. (1.3×10^{-6} m). The annular foot bears directly on the soil, supporting the weight of the GeoGauge™. Attached above this annular footing is a shaker, which excites the footing in a vertical mode. Sensors attached to the shaker and footing measure the force and displacement time histories.

The vertical excitation produces a small vertical harmonic excitation resulting in a small harmonic deflection at the soil-footing interface. The measured soil deflection ***d*** and the applied force *F* can be in turn used to calculate the stiffness of the underlying granular media. The problem of a rigid annular ring on a linear elastic, homogeneous, isotropic half space has been considered by Egorov (1965). The static stiffness *K* of such a soil-structure interaction problem has the functional form

$$K = \frac{F}{d} = \frac{ER}{(1-n^2)w(n)} \qquad (1)$$

where *E* and ***n*** are the modulus of elasticity and Poisson's ratio of the elastic media, respectively, *R* is the outside radius of the annular ring, and *w(n)* is a function of the ratio of the inside diameter and the outside diameter of the annular ring. For the ring geometry of the GeoGauge™, ω(n) is equal to 0.565. Recall that the modulus of elasticity and the modulus of rigidity G are relate through $G = \frac{E}{2(1+n)}$, hence, the above equation for static stiffness can be expressed as

$$K = \frac{F}{d} = \frac{3.54GR}{(1-n)} \qquad (2)$$

The ATR Institute at the University of New Mexico has been recently asked, by the New Mexico State Highway and Transportation Department (NMSHTD), to evaluate non-nuclear methods for the evaluation of compaction control of subgrade, subbase, and base course materials used in highway and transportation construction. The Humboldt GeoGauge™ was a identified as a potential alternative for such nuclear methods. While somewhat outside the scope of this paper, it can be argued that soil stiffness is a more fundamental property of the soil's ability to support surface loads than density measurements obtained by compaction methods. Most rational pavement and foundation design methods use stiffness or elastic modulus for the thickness determination of each layer of a pavement system.

The fundamental question that arises with any new technology is whether the new technology truly measures what the manufacturer claims. To this end the ATR Institute has embarked on an evaluation process of the GeoGauge™. This paper describes a simple laboratory experiment to determine if the GeoGauge™ truly measures stiffness as advertised.

Experiment

Dynamic soil-structure interaction experiments were conducted by Lenke, et al (1991), at the University of Colorado, using model footings in an enhanced gravitational field using the geotechnical centrifuge modeling technique. Because of the inability to truly model an elastic half space experimentally, they evaluated numerous container shapes and boundary materials to minimize reflected wave energy in an attempt to approximate true radiation damping of a vertically excited circular footing. One of the principal conclusions from their research was that cubical containers with a compliant energy absorbing boundary material allowed reasonable approximation of an elastic half space.

In order to evaluate the GeoGauge™, a similar approach was taken, albeit, on a larger scale than the experiments conducted by Lenke, et al. A relatively large steel box was obtained of approximate cubical shape. This steel container is lined with steel plate and reinforced externally by steel channels sections, resulting in a fairly rigid container. The nominal dimensions of this container are 24 in. (610 mm) deep with a lateral cross sectional area of 30 in. by 30 in. (760 mm by 760 mm). The lateral and bottom surfaces of this container were lined with 3/4 in. (19 mm) styrofoam panels as an energy absorbing material.

A dry granular cohesionless silica sand was obtained from U.S. Silica's Ottawa, Illinois manufacturing facility. The sand selected is designated as F-52. The product information provided by U.S. Silica state that this sand has a specific gravity of 2.65 (typical of silica sand) with a round grain shape. The mean grain size is approximately 0.26 mm (0.010 in.) based on a tabulated particle size distribution.

This silica sand was then pluviated through air from a height of 18 in. (460 mm) into the foamed lined steel test bin described above. This "raining" operation took approximately 20 hours to accomplish in order to ensure a uniform, highly compact granular media within the test bed. Upon completion of the raining operation, the surface of the sand media was carefully screeded level with the top of the test bin. During this operation the total weight of material placed was carefully tracked. Based on careful measurement of the test bin dimensions prior to pluviation of the sand, the resultant density of the media within the test bin could be calculated. The average density (unit weight) computed was 110.45 lb/ft^3 (1769.3 kg/ m^3). Based on this unit weight and the known specific gravity the void ratio of the granular media within the test bin was computed as 0.497.

With the granular media now in place within the test bin, measurements of stiffness were obtained using the Humboldt GeoGauge™. Measurements were obtained at the center of the test bin surface as well as at quarter points along one diagonal of the square cross section. A total of eight measurements were obtained with a mean value of 6.19 MN/m (35,300 lb/in.). The standard deviation of these eight measurements was 0.04 MN/m. The associated coefficient of variation as defined by the ratio of the standard deviation to the mean was 0.7%.

Analysis

Harding and Richart (1963) found that the modulus of rigidity (shear modulus) could be related to the void ratio and the mean effective octahedral stress (i.e., average effective confining pressure) by the following empirical equation.

$$G = \frac{2630(2.17-e)^2}{(1+e)}\left(\bar{s}_o\right)^{0.5} \qquad (3)$$

where G is the shear modulus, e is the void ratio of the granular media and $\bar{s}_o$ is the mean effective confining pressure. The above equation was developed for round-grained sands using dynamic wave propagation experimental methods. The above equation is empirical and has non-homogeneous units. The engineering units of both the shear modulus and mean effective stress are in terms of pounds per square inch (psi) and apparently the numerical value 2630 has units of $(\text{psi})^{0.5}$.

Equation (3) can be used to calculate the stiffness defined in Equation (2) if the unknown parameters can be estimated with some certainty. The void ratio in Equation (3) was very carefully determined during the experimental placement of the sand in the soil test bin. The value of the mean effective stress is much more difficult to ascertain, however. In addition, Poisson's ratio in Equation (2) is not known. However, if one can estimate the values of the mean effective stress and Poisson's ratio, then Equations (2) and (3) can be used to estimate the stiffness of the granular soil in the test bin and a comparison can be made with the GeoGauge™ experiment.

The mean effective stress can be computed using the following definition

$$\bar{s}_o = \frac{\bar{s}_v + 2\bar{s}_h}{3} = \frac{\bar{s}_v + 2K_o\bar{s}_v}{3} = \frac{\bar{s}_v}{3}\left(1 + 2K_o\right) \tag{4}$$

where $\bar{s}_v$ and $\bar{s}_h$ are the vertical and horizontal effective stress components, respectively, and K_o is the coefficient of lateral earth pressure. The value of K_o can be estimated by an equation developed by Jaky (1944)

$$K_o = \left(1 + \frac{2}{3}\sin f\right)\left(\frac{1 - \sin f}{1 + \sin f}\right) \tag{5}$$

where f is the effective angle of internal friction of the granular media. An estimate of this angle of internal friction was obtained by a simple experiment to determine the angle of repose of the F-52 silica sand used in the bin test described previously. The angle of repose measured was 33°. This is considered a lower bound for f; the actual value of f may approach 40° but the lower bound will be used for further computational analysis. Using the angle of repose as an approximation for the internal angle of friction yields a coefficient of lateral earth pressure of 0.402. Substitution of this value into Equation (4) yields the following for the mean effective stress

$$\bar{s}_o = 0.601\ \bar{s}_v \tag{6}$$

It can be shown that Poisson's ratio can be estimated using generalized Hooke's law as follows

$$n = \frac{K_o}{1 + K_o} \tag{7}$$

For the coefficient of lateral earth pressure previously estimated, the value of Poisson's ratio is calculated to be 0.287.

At this point, rational means have been used to estimate all variable for computing the stiffness of the silica sand used in the bin tests with the exception of the vertical effective stress. The value of this vertical effective stress is much more difficult to estimate. The effective stress below the annular footing can be considered as composed of two components. One component is the geostatic, or lithostatic stress caused by the self-weight of the material. This vertical self-weight component is simply the density (or unit weight) of the material γ times the depth below the footing, z. Essentially, the self weight component of vertical effective stress is zero at the ground surface of the soil bin and increases in a linear fashion with depth. The second component of vertical effective stress is caused by the presence of the annular footing on the surface of the experimentally modeled half space.

The analytical solution for the vertical stress distribution on centerline below an annularly loaded footing (r = 0) is presented in Poulos and Davis (1974) as

$$s_v = \frac{3pz^3 a}{\left(a^2 + z^2\right)^{5/2}} \qquad (8)$$

where a is the distance from the center of the footing to the centerline of the ring (see Figure 1), z is the depth below the centerline of the annular footing, and p is an annular line load acting at a distance a from the centerline of the footing. For the GeoGauge™ used in the experiments

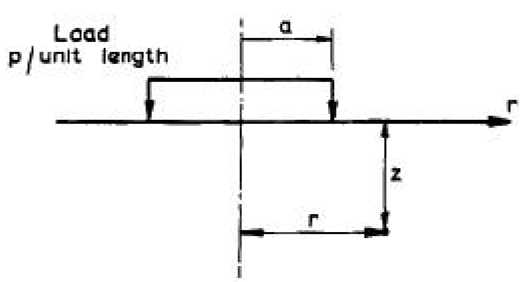

Figure 1. Uniform Vertical Ring Loading on Surface of Elastic Half Space.

described previously, a is equal to 2.0 in. (51 mm), and the weight of the GeoGauge™ was measured as 22.01 lb resulting in an annular line load p of 1.752 lb/in. (306.8 N/m). Note that the above equation is equal to the vertical effective stress as previously defined since the granular media as used in the experiments is dry and pore pressures are non existent.

Figure 2 shows a graphical representation of the vertical stress distribution as a function of depth for both the geostatic stress component and the annular ring induced component. The sum of this two stress components is the total vertical effective stress. The total stress distribution clearly shows that the stress levels become fairly constant and uniform for depths of 2 to 9 inches (50 mm to 230 mm). It is well known that the "pressure bulb" below an circular footing extends to a depth equal to about twice the diameter of the footing. The dynamic response of the annular footing will also be influenced by a zone of soil to a depth of about two diameters as well. Based on the observed total stress distribution of Figure 2 and a knowledge that the depth of influence extends to two diameters (9 in.), an estimate of 0.63 psi is made for the vertical effective stress below the annular footing.

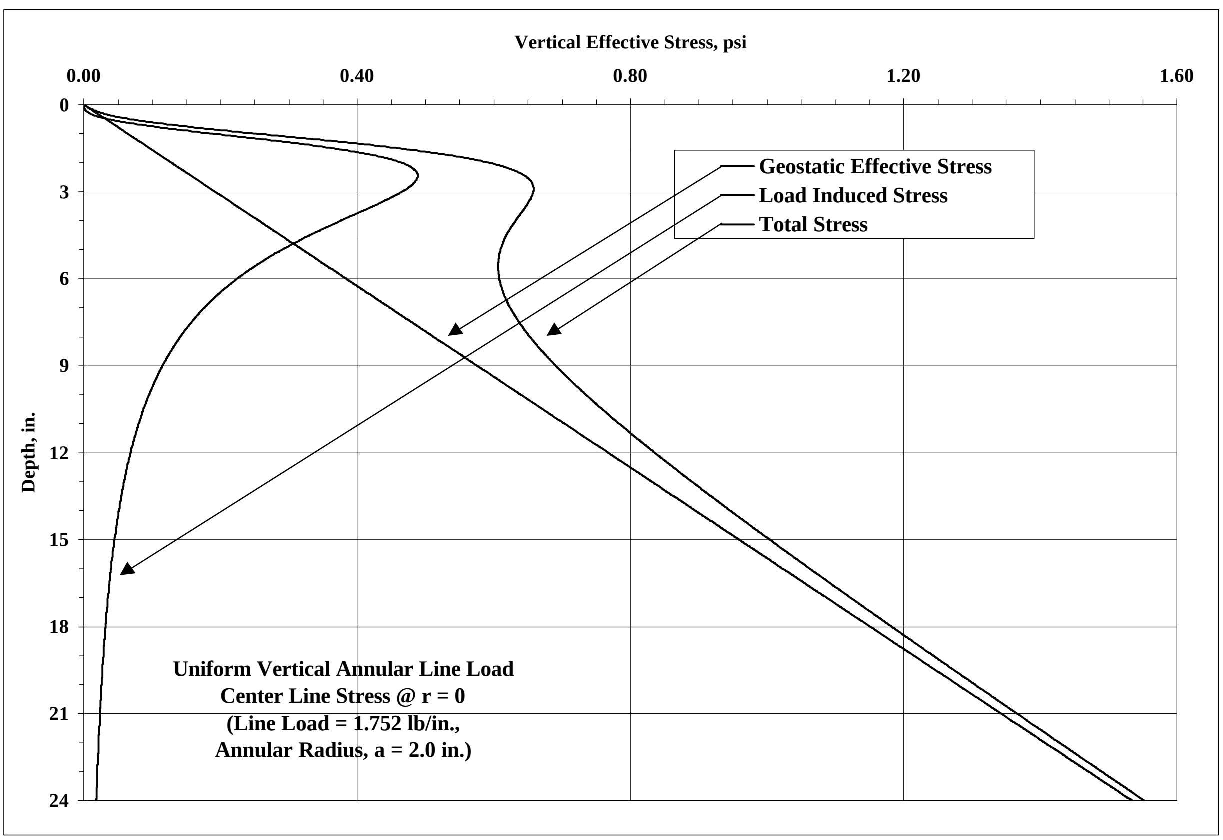

Figure 2. Effective Vertical Stress Distribution Below an Annular Footing.

Substitution of this estimated 0.63 psi vertical effective stress into Equation (6) with subsequent substitution of the mean effective stress into Equation (3) yields an estimate for the shear modulus of the granular media within the soil test bin. This shear modulus along with the previously estimated value of Poisson's ratio and the outside radius of the GeoGauge™ annular footing is then substituted into Equation (2) yielding a static stiffness of 33,800 lb/in.

Comparison of this computational estimate with the experimentally determined value of stiffness results in an error less than 5%. Based on this simple experiment and a rational analysis, one would conclude that the GeoGauge™ is indeed measuring the stiffness of the underlying granular soil media.

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